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Elastic Wave Velocities in Anisotropic Sedimentary Rocks

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Abstract

Elastic wave velocities and elastic constants of shale and clay slate are measured using the ultrasonic pulse method. Five independent elastic constants can be determined by three experiments using two samples with different directions. It is concluded that dilatational wave velocity along the direction of sedimentation is greater than that along the horizontal, and that shear wave velocity along the horizontal with particle motion in the direction of the horizontal is smaller than that with particle motion in the vertical direction ($SV<SH$).

1. Introduction

Many measurements of velocities of elastic waves in sedimentary rocks by the pulse method have been published thus far, but only a few have been reported of all their independent elastic constants. One of the reasons for this is due to the rather troublesome measuring techniques.

Even in the simplest case of a sedimentary rock, which is of axial symmetry, there are five independent elastic constants. Four of these constants can be measured without difficulty but the fifth, C_{13}, is usually obtained by measurements of elastic wave velocity along a special direction. Moreover the difference in the paths between wave and energy flow tenders the method complicated and ambiguous. The present study suggests a new and simple method for measurement of C_{13} in axially symmetric rocks (hexagonal symmetry). The method originally found by D. S. Hughes[1] for isotropic solids is to make use of a secondarily arrived wave besides a first arrived wave (dilatational wave).

In what follows, it will be shown that the method can be applied in the case of anisotropic solids. The results described here will also show the relation of velocities of elastic waves to the direction of sedimentation.

2. Arrival times of elastic pulses in an axial symmetry

Let us consider a plate with axial symmetry, whose surfaces are parallel to the c-axis (z-axis in crystallographic representation). If the plate is elastic and dissipationless, then the velocities of the elastic waves may be written[2),3)] as

$$\begin{vmatrix} \lambda_{11}-\rho V^2 & \lambda_{12} \\ \lambda_{12} & \lambda_{22}-\rho V^2 \end{vmatrix} = 0, \tag{1}$$

$$\lambda_{11} = C_{11} l^2 + C_{44} n^2$$

$$\lambda_{12}=(C_{13}+C_{44})ln$$
$$\lambda_{22}=C_{33}n^2+C_{44}l^2, \tag{2}$$

where V is the velocity of the elastic wave in the direction $(l,\ 0,\ n)$ and C_{11}, C_{13}, C_{33}, and C_{44} are elastic constants and ρ is the density. When $l=0$, $n=1$, we obtain the velocity of the dilatational wave along the z-axis,

$$V_p=\sqrt{\frac{C_{33}}{\rho}}. \tag{3}$$

The arrival time T_p of the dilatational wave is given by

$$T_p=L/V_p, \tag{4}$$

where L is the length of the plate in the z-axis.

In almost all materials, the velocity of the dilatational wave is greater than any other wave, so the above equation represents the first arrival wave. Another wave which follows the first arrival wave is the wave shown in Fig. 1, with the path OABO′. The wave propagates along OA as a dilatational wave* and changes into a rotational wave* at A and propagates along AB as a rotational and at B changes again into a dilatational and arrives at O′. The wave OABO′ was discovered by D. S. Hughes[1], and the existence of the same kind of wave in cubic crystals was reported[4]. As shown in Fig. 1, the angle φ_c between AB and the x-axis is the critical angle and satisfies[5],

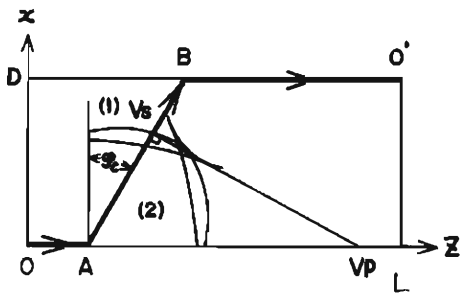

Fig. 1. Path of the wave. OABO′ indicates path of the second-arrival-wave. (1) and (2) are the velocity and wave surface respectively.

$$\frac{\sin 90^\circ}{V_p}=\frac{\sin\varphi_c}{V_s}, \tag{5}$$

where V_s is the velocity of the rotational* wave along AB. As V_s satisfies also Eq. (1) when $l=\cos\varphi_c$, and $n=\sin\varphi_c$, we obtain

$$\begin{vmatrix} \lambda_{11}-\rho V_p^2\sin^2\varphi_c & \lambda_{12} \\ \lambda_{12} & \lambda_{22}-\rho V_p^2\sin^2\varphi_c \end{vmatrix}=0. \tag{6}$$

The arrival time T_s of the wave OABO′ is given by

$$T_s=\frac{\mathrm{OA}+\mathrm{BO'}}{V_p}+\frac{\mathrm{AB}}{V_s}, \tag{7}$$

and the difference between the arrival times of the first and second waves is written as

$$t_d=T_s-T_p=\frac{D}{V_p\tan\varphi_c}, \tag{8}$$

* Strictly speaking, the wave travels as a quasi-dilatational or quasi-rotational wave.

where D is the thickness of the plate in the x-axis. Substituting Eq. (8) into (6), we obtain

$$\begin{vmatrix} \lambda_{11}-\rho V_p^2\dfrac{D^2}{V_p^2t_d^2+D^2} & \lambda_{12} \\ \lambda_{12} & \lambda_{22}-\rho V_p^2\dfrac{D^2}{V_p^2t_d^2+D^2} \end{vmatrix}=0. \tag{9}$$

After solving the above equation, we obtain

$$t_d=\sqrt{\frac{(C_{33}C_{44}+2C_{13}C_{44}+C_{13}^2)\rho}{C_{11}C_{33}C_{44}}}D. \tag{10}$$

The arrival time of the second wave T_e is given by

$$T_e=\frac{L}{V_p}+t_d. \tag{11}$$

The above calculation is also valid in the case of a plate in two dimensional x-y plane with free surfaces on the x-z plane. Then the following results can be easily obtained,

$$T_e=\frac{L}{V_p}+t_d, \tag{12}$$

$$V_p=\sqrt{\frac{C_{11}}{\rho}}, \tag{13}$$

$$t_d=\sqrt{\frac{(C_{11}+C_{12})\rho}{C_{11}(C_{11}-C_{12})}}D, \tag{14}$$

where D is the thickness of the plate in the direction of the x-axis.

3. Experiment and Results

The rock samples used were sedimentary rocks with very clearly parallel sedimentation. The samples showed very little anisotropy with respect to the direction perpendicular to sedimentation. Velocities of elastic waves along different directions in the x-y plane were almost constant within one percent. Therefore the rock sample can be assumed as an axial symmetry (hexagonal structure).

The shapes of the samples were parallel piped and the directions are shown in Fig. 2 in relation to the direction of sedimentation. In Table 1, dimensions of the samples are shown in relation to the crystallographic axis.

Using two samples with different directions as shown in Fig. 2, the following three measurements of elastic pulses were made.

1) Velocity of rotational wave along the direction of sedimentation was measured using shear mode barium titanate crystals of 500 kc/sec fund-

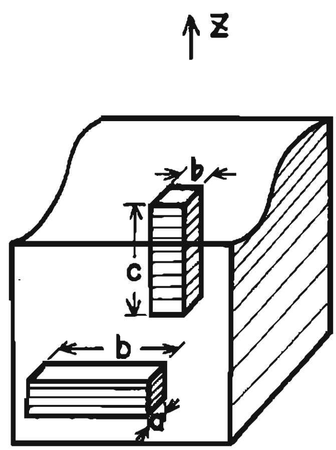

Fig. 2. Direction of the rock samples refered to the direction of sedimentation (z-axis).

TABLE 1.
Dimensions of the rock samples.

Sample No.	Rock	a (cm)	b (cm)	c (cm)	ρ (g/cm^3)
1	Shale	0.6159	0.6160	2.5388	2.724
3	Shale	0.5273	2.5388	0.6858	
5	Clay Slate	0.6199	0.6223	2.650	2.646
8	Clay Slate	0.4790	2.5352	0.7171	

a: length in x-axis (the direction perpendicular to sedimentation).
b: length in y-axis (the direction perpendicular to sedimentation and perpendicular to x-axis)
c: length in z-axis (the direction of sedimentation)

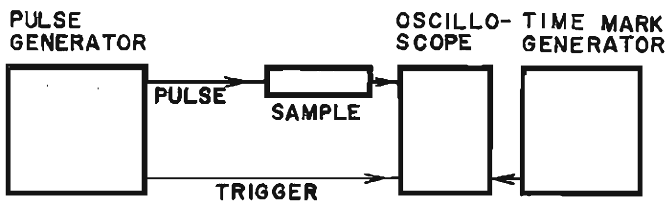

Fig. 3. Block diagram of the pulse experiment.

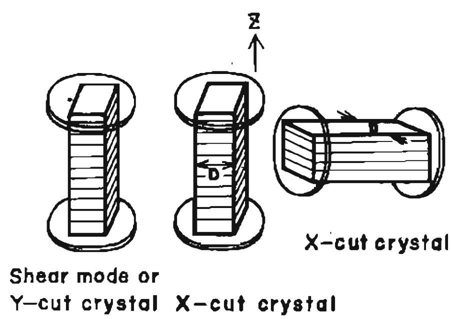

Fig. 4. Attachments of the piezoelectric crystals.

mental frequency or Y-cut quartz crystal of 5 Mc/sec.

2) Arrival times of the first and second elastic wave for both samples shown in Fig. 2 were measured using X-cut quartz crystals of 10 Mc/sec fundamental frequency.

In Fig. 3, and Fig. 4 the block diagram of the apparatus and attachments of the crystals are shown respectively.

From 1) we obtain $\sqrt{C_{44}/\rho}$ and from the first pulse in 2) both $\sqrt{C_{11}/\rho}$ and $\sqrt{C_{33}/\rho}$ are obtained. From the second pulses in 2) C_{12} and C_{13} are determined using Eqs. (14) and (10).

Typical records of pulses stated in 2) are shown in Fig. 5. The rocks used were a shale and a clay slate with parallel sedimentation. Colors of the shale and clay slate were black and yellow respectively. In Table 2, velocities of elastic waves and elastic constants are shown.

The values given in Table 2 are very large compared with other data previously published[6], but properties of sedimentary rocks vary very widely because of their past history.

It will be revealed that these rocks had been compressed by considerable pressure during their formation.

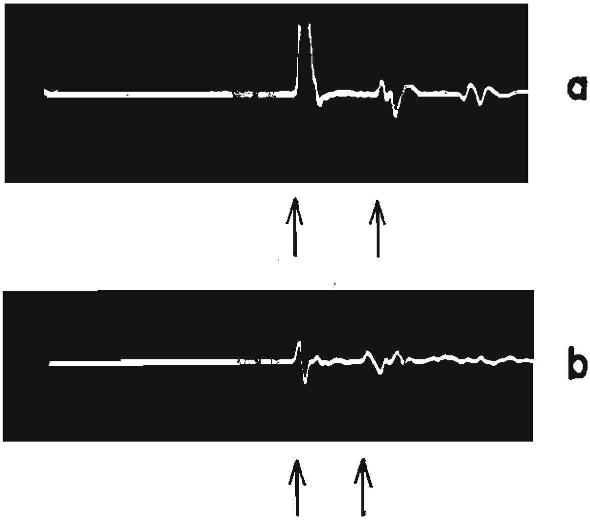

Fig. 5. Typical records of pulses. The arrows indicate arrival times of the first and second pulses. a and b indicate the sample No. 1 and 3 respectively.

TABLE 2.
Elastic wave velocities and elastic constants of rocks.

km/sec	V_1	V_2	V_3	V_4
Shale	5.42	3.40	5.56	3.03
Clay Slate	5.81	3.12	6.06	2.88

$\times 10^{11}$ dyne/cm^2	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}
Shale	7.99	1.69	1.82	8.42	2.50
Clay Slate	8.95	3.80	3.70	9.71	2.20

V_1: Velocity of dilatational wave in the direction perpendicular to that of sedimentation, $\sqrt{C_{11}/\rho}$.

V_2: Velocity of shear wave in the direction perpendicular to sedimentation with particle motion in the horizontal line. (SH), $\sqrt{(C_{11}-C_{12})/2\rho}$.

V_3: Velocity of dilatational wave in the direction of sedimentation, $\sqrt{C_{33}/\rho}$.

V_4: Velocity of shear wave in the direction perpendicular to sedimentation with particle motion in the direction of sedimentation (SV)=velocity of shear wave in the direction of sedimentation, $\sqrt{C_{44}/\rho}$.

C_{11}, ... : z-axis in the direction of sedimentation.

4. Conclusion

From Table 2, we can conclude that:

1) Velocity of dilatational wave along the direction of sedimentation is greater than that along the perpendicular direction.

2) Velocities of rotational wave along the direction of sedmentation with

particle motion in the direction perpendicular to sedimentation is greater than that with particle motion in the direction of sedimentation ($SH>SV$).

And with regard to the method for measuring elastic constants:

3) Using two samples with different directions, three measurements can be made to determine all five independent elastic constants for axially symmetrical rocks.

References

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